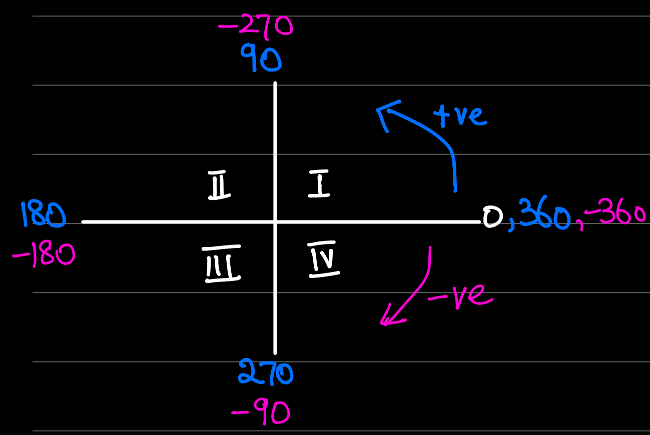


Sin +	All +ve
Tan +	Cos +

Angles are measured in two units

DEGREE

RADIAN

$$\boxed{180 \text{ degrees} = \pi \text{ radians}}$$

Famous angles to be memorized in terms of π

$90^\circ \longrightarrow \frac{\pi}{2}$

$45^\circ \longrightarrow \frac{\pi}{4}$

$30^\circ \longrightarrow \frac{\pi}{6}$

$60 \longrightarrow \frac{\pi}{3}$

$360 \longrightarrow 2\pi$

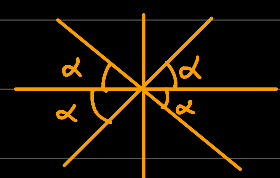
Syllabus : Recall these :

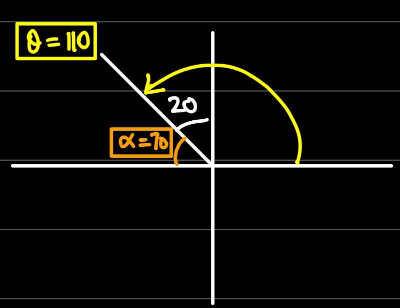
Degrees	0	30	45	60	90
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ infinite.

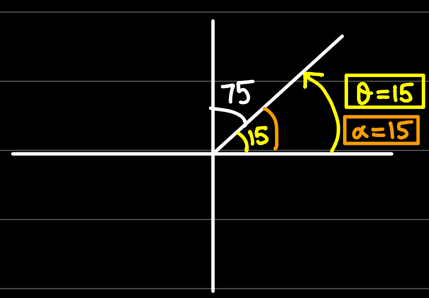
HOW TO MEASURE AN ANGLE (θ)

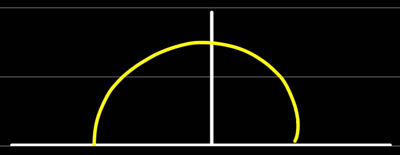
EVERY ANGLE HAS TWO MAIN VALUES

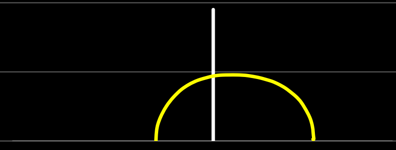
1) Original angle (θ)
2) Basic angle (α) = Acute angle with x-axis.

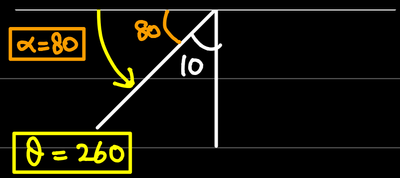


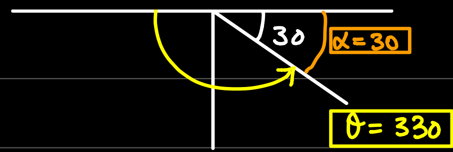












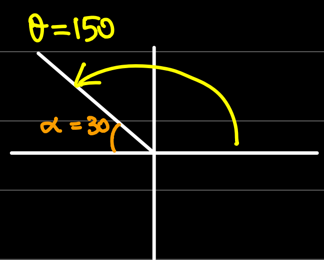
(sin/cos/ tan)

$$\begin{bmatrix}\text{TRIG RATIO OF} \\ \text{ANY ANGLE}(\theta)\end{bmatrix} = \begin{bmatrix}\text{TRIG RATIO OF} \\ \text{ITS BASIC ANGLE}(\alpha)\end{bmatrix}$$

AFTER ADUSTING +/− SIGNS FOR QUADRANTS.

Q. WITHOUT USING CALCULATOR, EVALUATE :

(i) $\sin 150$



$\sin 150$

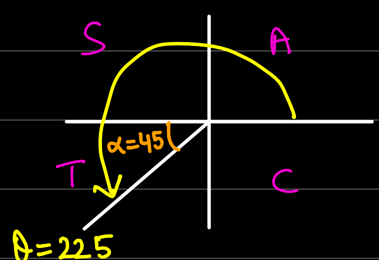
↓

$\sin 30 = \frac{1}{2}$

↓

$\boxed{\sin 150 = +\frac{1}{2}}$

(ii) $\cos 225 \longrightarrow \cos 45 = \frac{1}{\sqrt{2}} \longrightarrow \boxed{\cos 225 = -\frac{1}{\sqrt{2}}}$

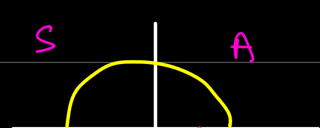


(iii) $\tan\left(\frac{11\pi}{6}\right)$ --> Super Important for P3 .

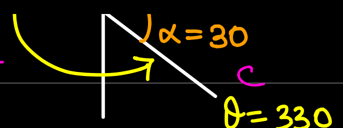
↓

$\tan\left(11 \times \frac{\pi}{6}\right)$ (30)

↓



$\tan(330)$



$\hookrightarrow \tan 30 = \frac{1}{\sqrt{3}} \longrightarrow \boxed{\tan 330 = \frac{-1}{\sqrt{3}}}$

$\boxed{\tan\left(\frac{11\pi}{6}\right) = \frac{-1}{\sqrt{3}}}$

420 — 5 (P1) & (P3)

5 — 6 (P3)

EQUATION SOLVING

QUADRANT?

Q: $\sin x = +\frac{1}{2}$ $\qquad 0 < x < 360$

① Permission to take inverse

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

③ Basic angle

② ignore any negative signs while taking inverse.

$\alpha = 30$

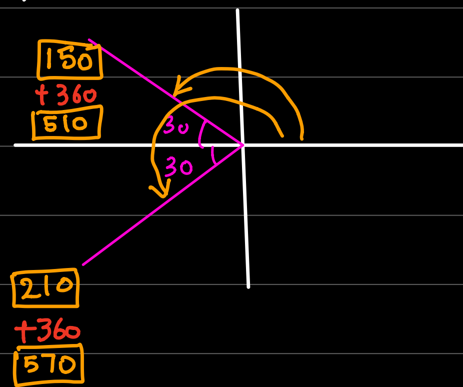
$x = 30, 150$

QUADRANT?

Q $\cos \underline{\underline{x}} = \dfrac{-\sqrt{3}}{2}$ $\quad 0 < \underline{\underline{x}} < 720$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

$$\boxed{x = 150, 210, 510, 570}$$

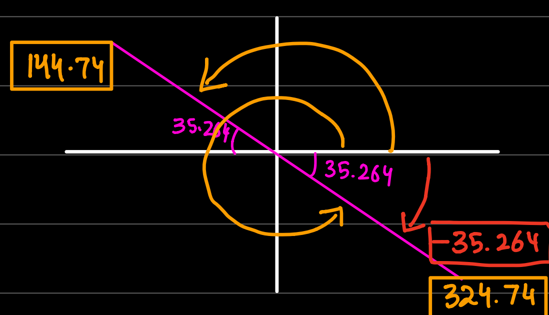


NOTE: IF RANGE IS MORE THAN 360, FIND FIRST TWO ANGLES AND KEEP ADDING 360 TO EACH ANSWER UNTIL IT GOES OUT OF GIVEN RANGE.

Q. $\tan \underline{\underline{x}} = -\dfrac{1}{\sqrt{2}}$ (Quadrant) $\quad -180 < \underline{\underline{x}} < 360$

$$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35.264$$

$$\boxed{x = -35.264, 144.74, 324.74}$$



V. IMP. RULE IF RANGE IS IN NEGATIVE ITS MANDATORY TO FIND NEGATIVE ANGLES FIRST.

IF NO PERMISSION TO TAKE INVERSE

RANGE CHANGE

Q. $\sin \underline{\underline{2x}} = -\frac{1}{2}$ $\qquad 0 < x < 360$

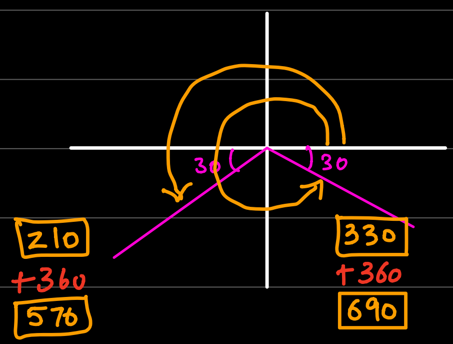
$\times 2$

$\boxed{2x = A}$ $\qquad 0 < \underline{\underline{2x}} < 720$

Quadrant

$\sin A = -\frac{1}{2}$ $\qquad 0 < A < 720$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30$



$2x = A = 210, 330, 570, 690$

$2x = 210, 330, 570, 690$

$\boxed{x = 105, 165, 285, 345}$

Q. $\cos(\underline{\underline{x - 70}}) = \frac{1}{2}$ $\qquad 0 < x < 360$

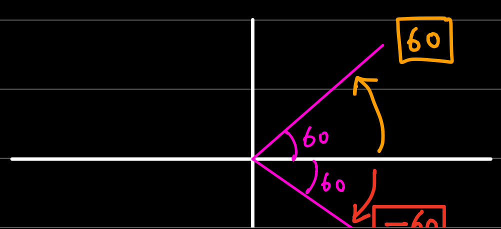
-70

$A = x - 70$ $\qquad -70 < \underline{\underline{x-70}} < 290$

Quad

$\cos A = +\frac{1}{2}$ $\qquad -70 < A < 290$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$



$A = -60, 60$

$x - 70 = -60, 60$

$x = 10, 130$

IDENTITIES (17 + 8 = 25)

(P1 + P3)

RECIPROCAL
$\frac{1}{\sin x} \equiv \operatorname{cosec} x$
$\frac{1}{\cos x} \equiv \sec x$
$\frac{1}{\tan x} \equiv \cot x$

$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$\sin^2 x = (\sin x)^2$

$\sin x^2 = \sin(x^2)$

$$\sin(180 - \theta) = \sin\theta$$
$$\cos(180 - \theta) = -\cos\theta$$
$$\tan(180 - \theta) = -\tan\theta$$

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

$$\sin(90 - \theta) = \cos\theta$$
$$\cos(90 - \theta) = \sin\theta$$
$$\tan(90 - \theta) = \frac{1}{\tan\theta}$$

P3 ONLY

DOUBLE ANGLE

$$\sin 2x \equiv 2\sin x \cos x$$

$$\cos 2x \equiv \cos^2 x - \sin^2 x$$

$$\equiv 2\cos^2 x - 1$$

$$\equiv 1 - 2\sin^2 x$$

$$\tan 2x \equiv \frac{2\tan x}{1 - \tan^2 x}$$

COMPOUND ANGLE

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

V.V. IMP DOUBLE ANGLE ADVANCED VARIATION.

$\sin 2x \equiv 2\sin x \cos x$	$\cos 2A \equiv 2\cos^2 A - 1$	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
$\sin 4x \equiv 2\sin 2x \cos 2x$	$\cos 4A \equiv 2\cos^2 2A - 1$	$\tan 4A \equiv \frac{2\tan 2A}{1-\tan^2 2A}$
$\sin 6x \equiv 2\sin 3x \cos 3x$	$\cos 6A \equiv 2\cos^2 3A - 1$	

COMPOUND ANGLE

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

1) $\sin(x+30) = \sin x \cos 30 + \cos x \sin 30$

$$= (\sin x)\left(\frac{\sqrt{3}}{2}\right) + \cos x \left(\frac{1}{2}\right)$$

2) $\cos(x-45) = \cos x \cos 45 + \sin x \sin 45$

$$(\cos x)\left(\frac{1}{\sqrt{2}}\right) + \sin x\left(\frac{1}{\sqrt{2}}\right)$$

3) $$\tan(x+60) = \frac{\tan x + \tan 60}{1 - \tan x \tan 60}$$

$$= \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}$$

$\boxed{\text{TYPE 1}}$ (i) Form $R\sin(\theta \pm \alpha)$ OR $R\cos(\theta \pm \alpha)$

(ii) Solve equation.

$\boxed{\text{CONCEPT}}$

$3\sin x - 4\cos x = 0$	$3\sin x - 4\cos x = 2$
$3\sin x = 4\cos x$	YOU CANNOT SOLVE THIS DIRECTLY.
$\frac{\sin x}{\cos x} = \frac{4}{3}$	
$\tan x = \frac{4}{3}$	
Find alpha:	

TYPE 1 QUESTION IS ALWAYS IN TWO PARTS.

(i) Express in form

(ii) Solve equation.

Q. (i) Express $3\sin\theta - 4\cos\theta$ in form $R\sin(\theta-\alpha)$ where $R>0$ and $0<\alpha<90$. (4 marks)

$$3\sin\theta - 4\cos\theta \equiv R\sin(\theta-\alpha)$$
$$\equiv R[\sin\theta\cos\alpha - \cos\theta\sin\alpha]$$
$$3\sin\theta - 4\cos\theta \equiv R\cos\alpha\sin\theta - R\sin\alpha\cos\theta$$
$$-R\sin\alpha = -4$$
$$R\sin\alpha = 4 \qquad R\cos\alpha = 3$$

STEP1:

$$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{4}{3}$$
$$\tan\alpha = \frac{4}{3}$$
$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$
$$\alpha = 53.13$$

STEP2: SQUARE BOTH AND ADD.

$$R^2\sin^2\alpha = 16$$
$$+\ R^2\cos^2\alpha = 9$$
$$R^2\sin^2\alpha + R^2\cos^2\alpha = 16+9$$
$$R^2(\sin^2\alpha+\cos^2\alpha) = 25$$
$$R^2(1) = 25$$
$$R^2 = 25$$
$$R = 5$$

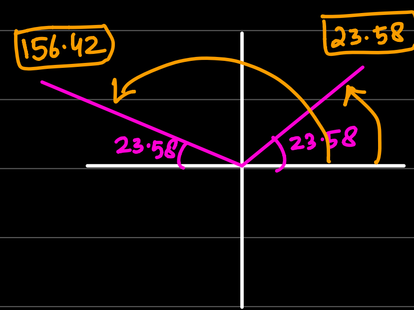
$$3\sin\theta - 4\cos\theta \equiv 5\sin(\theta-53.13)$$

(ii) Hence Solve $3\sin\theta - 4\cos\theta = 2$ for $0<\theta<360$ (4 marks)

$$5\sin(\theta-53.13) = 2 \qquad 0<\theta<360$$
$$-53.13$$
$$\sin(\theta-53.13) = 0.4$$
$$-53.13<\theta-53.13<306.87$$
$$\boxed{\theta-53.13 = A}$$
$$\sin A = +0.4 \qquad -53.13<A<306.87$$

$\alpha = \sin^{-1}(0.4)$

$\alpha = 23.58$



$A = 23.58, \ 156.42$

$\theta - 53.13 = 23.58, \ 156.42$

$\theta = 76.71, \ 209.55$

2 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

9709/03/M/J/06

$7\cos\theta + 24\sin\theta \equiv R\cos(\theta - \alpha)$

$R[\cos\theta\cos\alpha + \sin\theta\sin\alpha]$

$7\cos\theta + 24\sin\theta \equiv R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\sin\alpha = 24 \qquad R\cos\alpha = 7$

$\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{24}{7}$

$\tan\alpha = \dfrac{24}{7}$

$\alpha = 73.74$

$R^2\sin^2\alpha = 24^2$

$R^2\cos^2\alpha = 7^2$

$R^2\sin^2\alpha + R^2\cos^2\alpha = 24^2 + 7^2$

$R^2(\sin^2\alpha + \cos^2\alpha) = 625$

$R^2(1) = 625$

$R = 25$

$7\cos\theta + 24\sin\theta \equiv R\cos(\theta - \alpha)$

$7\cos\theta + 24\sin\theta \equiv 25\cos(\theta - 73.74)$

(ii) $7\cos\theta + 24\sin\theta = 15$ $\qquad 0 \le \theta \le 360$

$25\cos(\theta - 73.74) = 15$ $\qquad -73.74$

$\cos(\theta - 73.74) = 0.6$ $\qquad -73.74 < \theta - 73.74 < 286.26$

$A = \theta - 73.74$

$\cos A = 0.6$ $\qquad -73.74 < A < 286.26$

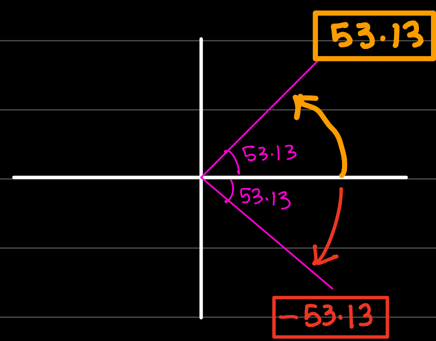
$\alpha = \cos^{-1}(0.6)$

$\alpha = 53.13$

$A = -53.13, 53.13$

$\theta - 73.74 = -53.13, 53.13$

$\theta = 20.61, 126.87.$



ADVANCED VARIATION (TRAP)

BE VERY VERY CAREFUL ABOUT THIS

Let's suppose this was result of first part.

$7\cos\theta + 24\sin\theta \equiv 25\cos(\theta - 73.74)$

For second part: Substitute in place of θ.

$7\cos 2x + 24\sin 2x \longrightarrow 25\cos(2x - 73.74)$

$7\cos\left(\frac{1}{2}\theta\right) + 24\sin\left(\frac{1}{2}\theta\right) \longrightarrow 25\cos\left(\frac{1}{2}\theta - 73.74\right)$

3 **(i)** Express $5\sin x + 12\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\sin 2\theta + 12\cos 2\theta = 11,$$

giving all solutions in the interval $0° < \theta < 180°$. [5]

9709/03/O/N/08

(i) $5\sin x + 12\cos x \equiv 13\sin(x + 67.38)$

(ii) $5\sin 2\theta + 12\cos 2\theta \equiv 13\sin(2\theta + 67.38)$

$\underbrace{5\sin 2\theta + 12\cos 2\theta}$ $= 11$

$13\sin(2\theta + 67.38) = 11$

$\sin(2\theta + 67.38) = \frac{11}{13}$

$0 < \theta < 180$

$\times 2$

$0 < 2\theta < 360$

$+67.38$

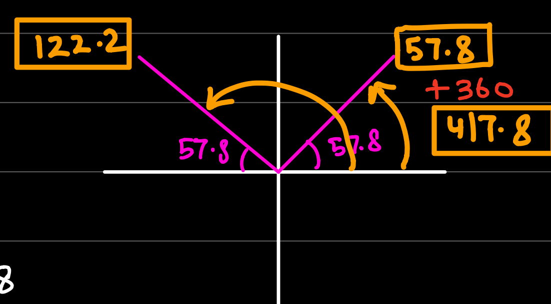
$67.38 < 2\theta + 67.38 < 427.38$

$A = 2\theta + 67.38$

$\sin A = \frac{11}{13}$

$67.38 < A < 427.38$

$\alpha = \sin^{-1}\left(\frac{11}{13}\right)$



$\alpha = 57.80$

Out of range

$A = \cancel{57.8},\ 122.2,\ 417.8$

$2\theta + 67.38 = 122.2,\ 417.8$

$2\theta = 54.82,\ 350.42$

$\theta = 27.41 \; , \; 175.21$

7 (i) Express $24\sin\theta - 7\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence find the smallest $\boxed{\text{positive value of } \theta}$ satisfying the equation

$$24\sin\theta - 7\cos\theta = 17.$$ [2]

You will choose the first answer.

Range?

9709/33/O/N/12

(i) $24\sin\theta - 7\cos\theta \equiv 25\sin(\theta - 16.26)$

$24\sin\theta - 7\cos\theta = 17$

$25\sin(\theta - 16.26) = 17$

$\sin(\theta - 16.26) = \dfrac{17}{25}$

$\theta > 0$

-16.26

$\theta - 16.26 > -16.26$

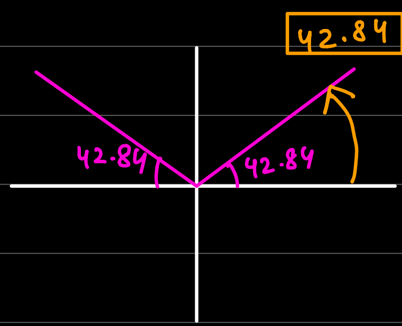
$A = \theta - 16.26$

$\sin A = \dfrac{17}{25}$

$A > -16.26$

$\alpha = \sin^{-1}\left(\dfrac{17}{25}\right)$

$\alpha = 42.84$



$A = 42.84$

$\theta - 16.26 = 42.84$

$\theta = 59.1$

4 **(i)** Express $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

$\theta > 0$

(a) $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta = -4$, [2]

(b) $(\sqrt{6})\cos\frac{1}{2}\theta + (\sqrt{10})\sin\frac{1}{2}\theta = 3$. [4]

(i) $\sqrt{6}\cos\theta + \sqrt{10}\sin\theta \longrightarrow 4\cos(\theta - 52.24)$

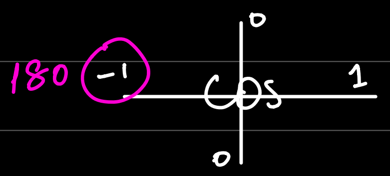
(a) $\sqrt{6}\cos\theta + \sqrt{10}\sin\theta = -4$

$4\cos(\theta - 52.24) = -4$ $\qquad \theta > 0$

$\cos(\theta - 52.24) = -1$ $\qquad \theta - 52.24 > -52.24$

$A = \theta - 52.24$

$\cos A = -1$ $\qquad A > -52.24$



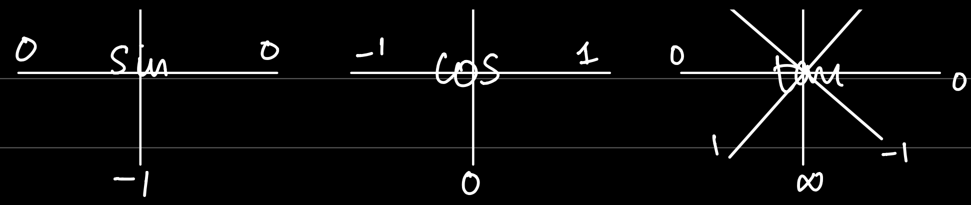
$A = 180$

$\theta - 52.24 = 180$

$\theta = 232.24$.

NEVER	DO	$\sin^{-1}(\ )$	for	0, 1, −1
		$\cos^{-1}(\ )$		
		$\tan^{-1}(\ )$		

instead, use grids.



(11) $\sqrt{6}\cos\theta + \sqrt{10}\sin\theta \longrightarrow 4\cos(\theta - 52.24)$

$\sqrt{6}\cos\left(\frac{1}{2}\theta\right) + \sqrt{10}\sin\left(\frac{1}{2}\theta\right) = 3$

$\theta > 0$

$\div 2$

$\frac{1}{2}\theta > 0$

-52.24

$\frac{1}{2}\theta - 52.24 > -52.24$

$4\cos\left(\frac{1}{2}\theta - 52.24\right) = 3$

$\cos\left(\frac{1}{2}\theta - 52.24\right) = 0.75$

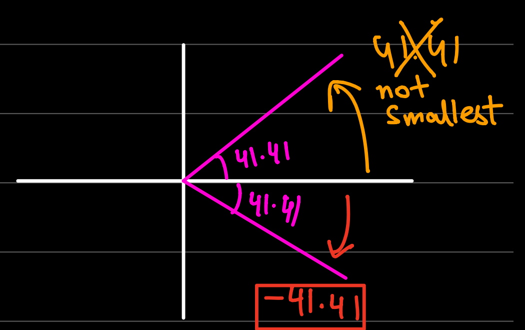
$A = \frac{1}{2}\theta - 52.24$

$\cos A = 0.75$

$A > -52.24$

$\alpha = \cos^{-1}(0.75)$

$\alpha = 41.41$



$A = -41.41$

$\frac{1}{2}\theta - 52.24 = -41.41$

$\theta = 21.7$

TYPE2 Compound angle formulas.

(Case1) sin and cos $\longrightarrow$ bring to tan.

This type usually does not contain RANGE CHANGE.

5 Solve the equation

$$\cos(\theta + 60^\circ) = 2\sin\theta,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

 9709/31/O/N/10

$\cos(\theta + 60) = 2\sin\theta$ $\quad 0 \leq \theta \leq 360$

$\cos\theta\cos 60 - \sin\theta\sin 60 = 2\sin\theta$

$\cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right) = 2\sin\theta$

$\frac{\cos\theta - \sqrt{3}\sin\theta}{2} = 2\sin\theta$

$\cos\theta - \sqrt{3}\sin\theta = 4\sin\theta$

$4\sin\theta + \sqrt{3}\sin\theta = \cos\theta$

$(4+\sqrt{3})\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{4+\sqrt{3}}$

Imp Keep everything in EXACT FORM.

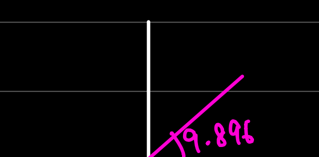
$\tan\theta = \underbrace{\frac{1}{4+\sqrt{3}}}$

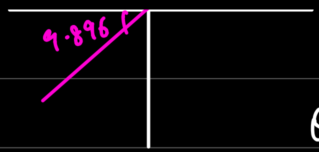
Note Find this numerical value before taking inverse. because this may be negative and

$\tan\theta = 0.1746$

$\alpha = \tan^{-1}(0.17746)$

$\alpha = 9.896$





$\theta = 9.896,\ 189.896.$

we can't take inverse on -ve.

8 Solve the equation

$$\sin(\theta + 45^\circ) = 2\cos(\theta - 30^\circ),$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

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$\sin(\theta + 45) = 2\cos(\theta - 30)$ $\quad 0 < \theta < 180$

$\sin\theta\cos 45 + \cos\theta\sin 45 = 2(\cos\theta\cos 30 + \sin\theta\sin 30)$

$\sin\theta\left(\frac{1}{\sqrt{2}}\right) + \cos\theta\left(\frac{1}{\sqrt{2}}\right) = 2\left(\cos\theta\left(\frac{\sqrt{3}}{2}\right) + \sin\theta\left(\frac{1}{2}\right)\right)$

$\frac{\sin\theta + \cos\theta}{\sqrt{2}} = \cancel{2}\left(\frac{\sqrt{3}\cos\theta + \sin\theta}{\cancel{2}}\right)$

$\sin\theta + \cos\theta = \sqrt{6}\cos\theta + \sqrt{2}\sin\theta$

$\sin\theta - \sqrt{2}\sin\theta = \sqrt{6}\cos\theta - \cos\theta$

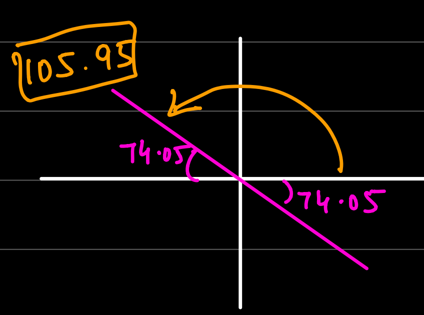
$(1 - \sqrt{2})\sin\theta = (\sqrt{6} - 1)\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{6} - 1}{\underbrace{1 - \sqrt{2}}_{\text{see this was negative}}}$

$\tan\theta = -3.49938$

$\alpha = \tan^{-1}(3.49938)$

$\alpha = 74.05$



$\theta = 105.95.$

Type 2: Case 2: COMPOUND ANGLE ON TAN.

Method: Make a QUADRATIC EQUATION & SOLVE.

2 (i) Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

 9709/03/O/N/07

$$\frac{\tan 45 + \tan x}{1 - \tan 45 \tan x} - \tan x = 2$$

$$\frac{1 + \tan x}{1 - \tan x} = 2 + \tan x$$

$$1 + \tan x = (2 + \tan x)(1 - \tan x)$$

$$1 + \tan x = 2 - 2\tan x + \tan x - \tan^2 x$$

$$\tan^2 x + \tan x + 2\tan x - \tan x + 1 - 2 = 0$$

$$\tan^2 x + 2\tan x - 1 = 0$$

(ii) Solve: $\tan^2 x + 2\tan x - 1 = 0$ $\quad 0 \leq x \leq 180$

$$\tan x = a$$

$$a^2 + 2a - 1 = 0$$

$$a = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$$

$$a = \frac{-2 \pm \sqrt{8}}{2}$$

$$a = \frac{-2+\sqrt{8}}{2} \qquad a = \frac{-2-\sqrt{8}}{2}$$

$$a = 0.4142 \qquad a = -2.4142$$

$\tan x = 0.4142$

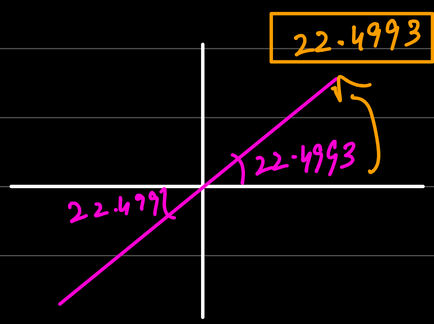
$\alpha = \tan^{-1}(0.4142)$

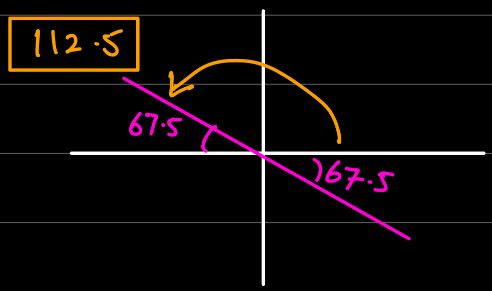
$\alpha = 22.4993$

$\tan x = -2.4142$

$\alpha = \tan^{-1}(2.4142)$

$\alpha = 67.5$





TYPE3 IDENTITY PROVING

IDENTITY PROVING

P3 TIP: KEEP AN EYE ON THE ANGLE
(DOUBLE ANGLE & COMPOUND ANGLE)

SQUARED TERMS

* $\sin^2\theta + \cos^2\theta = 1$
* $1 + \tan^2\theta = \sec^2\theta$
* $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
* $a^2 - b^2 = (a+b)(a-b)$

NO SQUARED TERMS

Bring everything to sin and cos ONLY and simplify.

You are only allowed to solve one side of identity and prove it equal to other side. You are not allowed to start simplifying both sides at the same time.

3 (i) Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta$. [3]

$2\theta \longrightarrow \theta$
Double angle

$$\frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$$

$$\frac{1 + \cos 2\theta}{\sin 2\theta}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\frac{\cancel{1} + 2\cos^2\theta \cancel{- 1}}{2\sin\theta\cos\theta}$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$$\frac{\cancel{2}\cos^{\cancel{2}}\theta}{\cancel{2}\sin\theta\cancel{\cos\theta}}$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

Easy identity (very Long).

11 Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

Compound angle

1 page

Double angle

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Apply compound angles instantly.

$$\frac{\tan 45 + \tan x}{1 - \tan 45 \tan x} + \frac{\tan 45 - \tan x}{1 + \tan 45 \tan x}$$

$$\frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x}$$

$$\frac{(1 + \tan x)^2 + (1 - \tan x)^2}{(1 - \tan x)(1 + \tan x)}$$

$$\frac{1 + \cancel{2\tan x} + \tan^2 x + 1 \cancel{- 2\tan x} + \tan^2 x}{1 - \tan^2 x}$$

$$\frac{2+2\tan^2 x}{1-\tan^2 x}$$

Here we decided to go for sin & cos and also that we will need double angle.

$$\frac{2+\frac{2\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{\frac{2\cos^2 x+2\sin^2 x}{\cos^2 x}}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

$$\frac{2\cos^2 x+2\sin^2 x}{\cos^2 x} \div \frac{\cos^2 x-\sin^2 x}{\cos^2 x}$$

$$\frac{2\cos^2 x+2\sin^2 x}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\cos^2 x-\sin^2 x}$$

$$\frac{2(\cos^2 x+\sin^2 x)}{\cos^2 x-\sin^2 x}$$

$$\frac{2(1)}{\cos 2x}$$

$\cos 2x = \cos^2 x - \sin^2 x$

$$\frac{2}{\cos 2x} = \boxed{2\sec 2x}$$

Another variation.

$$\frac{1+\tan x}{1-\tan x} + \frac{1-\tan x}{1+\tan x}$$

$$\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} + \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$$

$$\frac{\frac{\cos x+\sin x}{\cancel{\cos x}}}{\frac{\cos x-\sin x}{\cancel{\cos x}}} + \frac{\frac{\cos x-\sin x}{\cancel{\cos x}}}{\frac{\cos x+\sin x}{\cancel{\cos x}}}$$

$$\frac{\cos x+\sin x}{\cos x-\sin x} + \frac{\cos x-\sin x}{\cos x+\sin x}$$

$$\frac{(\cos x+\sin x)^2+(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$\frac{\cos^2 x+\cancel{2\sin x\cos x}+\sin^2 x+\cos^2 x-\cancel{2\sin x\cos x}+\sin^2 x}{\cos^2 x-\sin^2 x}$$

$$\frac{2\cos^2 x+2\sin^2 x}{\cos^2 x-\sin^2 x} = \frac{2(\cos^2 x+\sin^2 x)}{\cos^2 x-\sin^2 x}$$

$$= \frac{2(1)}{\cos 2x}$$

$$= 2\sec 2x.$$

FOR ADVANCED TRIG IDENTITIES

$4\theta \longrightarrow \theta$

$4\theta \xrightarrow[\text{identity}]{\text{Double angle}} 2\theta \xrightarrow[\text{identity}]{\text{Double angle}} \theta$

$3\theta \longrightarrow \theta$

write 3θ as $(2\theta + \theta)$

↓

Apply compound angle to separate 2θ and θ

↓

Apply Double angle identity on 2θ terms.

5 (i) Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 4\theta = 2\cos^2 2\theta - 1$

$\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$

$2\cos^2 2\theta - 1 + 4(2\cos^2\theta - 1)$

$2(\cos 2\theta)^2 - 1 + 8\cos^2\theta - 4$

$2(2\cos^2\theta - 1)^2 - 1 + 8\cos^2\theta - 4$

$2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 + 8\cos^2\theta - 4$

$8\cos^4\theta - \cancel{8\cos^2\theta} + 2 - 1 + \cancel{8\cos^2\theta} - 4$

$8\cos^4\theta - 3$ (Q.E.D)

7 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

$\cos 4\theta - 4\cos 2\theta$ $\qquad \cos 2\theta = 1 - 2\sin^2\theta$

$1 - 2\sin^2 2\theta - 4(1 - 2\sin^2\theta)$ $\qquad \cos 4\theta = 1 - 2\sin^2 2\theta$

$1 - 2(\sin 2\theta)^2 - 4 + 8\sin^2\theta$

$1 - 2(2\sin\theta\cos\theta)^2 - 4 + 8\sin^2\theta$

$1 - 2(4\sin^2\theta\cos^2\theta) - 4 + 8\sin^2\theta$

$1 - 8\sin^2\theta\cos^2\theta - 4 + 8\sin^2\theta$

$1 - 8\sin^2\theta(1 - \sin^2\theta) - 4 + 8\sin^2\theta$

$1 \cancel{- 8\sin^2\theta} + 8\sin^4\theta - 4 \cancel{+ 8\sin^2\theta}$

$8\sin^4\theta - 3$ (Q.E.D)

6 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \qquad [4]$$

$\sin(2\theta + \theta)$

Apply compound angle

$\sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$(2\sin\theta\cos\theta)\cos\theta + (1 - 2\sin^2\theta)\sin\theta$

$2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$3\sin\theta - 4\sin^3\theta$